Q1) [32 pts] Circle the correct answer.

(1) $\lim_{x \to \infty} (\ln(2x) - \ln(x+1)) =$

(a) 0

(b) 2

(c) $\ln 2$

(d) ∞

$\lim \ln\left(\frac{2x}{x+1}\right) =$

$\frac{\ln 2x}{\ln x+1}$

(2) $\int_0^{\pi/2} 14 \cos(3x) \cos(4x)\, dx =$

(a) 0

(b) 8

(c) 6

(d) -6

$14\int \cos(3x)\cos(4x)\,dx$

$\frac{14}{2}\int \cos(-x) + \cos(7x)$

$7\int \cos(-x) + \int \cos 7x$

$7\frac{\sin(-x)}{-1} + \frac{7\sin 7x}{7}$

$-7\sin(-x) + \sin 7x$

(3) If the mass of Polonium decreases according to the equation $y = y_0 e^{kt}$ and the half-life of Polonium is $\ln(16)$ minutes, then the decay rate $k =$

(a) -4

(b) 4

(c) $\frac{1}{4}$

(d) $-\frac{1}{4}$

$y = y_0 e^{kt}$

(4) If $f(x) = e^{\tan^{-1}(2x)}$, then $f'(0) =$

(a) 2

(b) $2e$

(c) 1

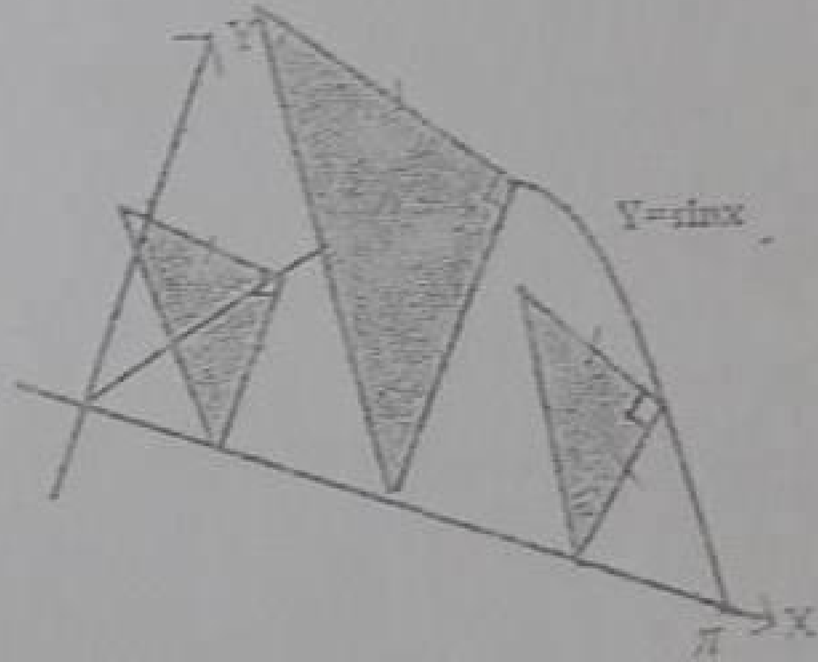
(d) e

$e^0 = 1$

Question#3 (12%)

a) find the length of the curve $f(x)=\frac{1}{3}(x^2+2)^{\frac{3}{2}}, 0 \le x \le 2$

b)The base of a solid is the region between the curve $y= 2\sqrt{\sin x}$ and the X- axis on the interval $[0\ \pi]$. The cross sections perpendicular to the X-axis are isosceles right triangles with one leg running from the X-axis to the curve. find the volume of the solid



19) If $g(x)=\int_{x}^{x^3}\frac{dt}{t}$ where $x>0$, then $g(x)' =$

a) $\frac{2}{x}$ b) $\frac{1}{x}$ c) $\frac{1}{x^3}-\frac{1}{x}$ d) $\frac{3}{x^3}-\frac{1}{x}$

20) The average value of the function $f(x)=2x\sqrt{x^2+1}$ on $[0,\sqrt{3}]$ is

a) $\frac{2}{\sqrt{3}}$ b) $\frac{7}{3\sqrt{3}}$ c) $\frac{14}{3\sqrt{3}}$ d) $\frac{8}{\sqrt{3}}$

21) If $f'(x)=x^2-x^4$ then $f(x)$ has

a) Local maximum at $x=-1$ and local minimum at $x=1$

b) Local maximum at $x=1$ and local minimum at $x=-1$

c) Local maximum at $x=0$ and local minimum at $x=1$

d) Local maximum at $x=1$ and local minimum at $x=0$

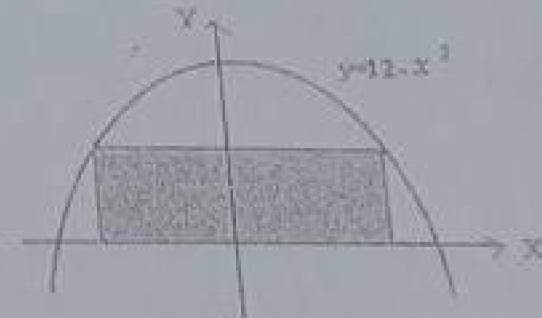
21) If the volume of a sphere is changing at the rate 4π ft^3/sec when the radius $r=2$ ft then the rate of change of the radius is

a) $\frac{1}{2}$ ft/sec b) $\frac{1}{4}$ ft/sec c) $\frac{3}{2}$ ft/sec d) $\frac{5}{2}$ ft/sec

16) If P is a partition for the interval $[0,\pi]$ into n equal subintervals , then $\lim_{n\to\infty}\sum_{k=1}^{n}\frac{\pi}{n}(\cos\frac{k\pi}{n}) =$

a) 0 b) 1 c) -1 d) Does not exist

17) The largest area of a retangle whith base on the X- axis and its upper two vertices on the parabola $y=12-x^2$ is



a) 24 b) 32 c) 16 d) 48

18) Let $f(x)=x^2$ defined on $[a,b]$, then the value of c that satisfies the mean value theorem is

a) $a+b$ b) $\frac{a-b}{2}$ c) $\frac{a+b}{2}$ d) $\sqrt{ab}$

19) If $g(x)=\int_{x}^{x^3}\frac{dt}{t}$ where $x>0$, then $g(x)'=$

a) $\frac{2}{x}$ b) $\frac{1}{x}$ c) $\frac{1}{x^3}-\frac{1}{x}$ d) $\frac{3}{x^3}-\frac{1}{x}$

20) The average value of the function $f(x)=2x\sqrt{x^2+1}$ on $[0,\sqrt{3}]$ is

a) $\frac{2}{\sqrt{3}}$ b) $\frac{7}{3\sqrt{3}}$ c) $\frac{14}{3\sqrt{3}}$ d) $\frac{8}{\sqrt{3}}$

12) If $\int_{3}^{1} f(x)dx=5$, and $\int_{1}^{2} f(x)dx=7$, then $\int_{2}^{3} f(x)dx=$

a) -12 b) 2 c) -2 d) 12

13) $\int_{0}^{1}\sqrt{1-x^2}\,dx =$

a) $\frac{\pi}{4}$ b) $\frac{\pi}{2}$ c) $\frac{\pi}{6}$ d) π

14) If $h(x)=\dfrac{2f(x)}{x+2\cos x}$, $g(x)=\sqrt{x-1}$ and $f(0)=2$ $f'(0)=-1$ then $(g\circ h)'(0)=$

a) 0 b) 1 c) -2 d) -1

15) If $f(x)=\sin^2(x)-2x$ then $f(x)$ has

a) At least two real roots b) At least one real roots

c) Exactly one real root d) Exactly two real roots

8) $\lim\limits_{x \to 2} \dfrac{x-2}{\sqrt{2x+5}-3} =$

a) [illegible] b) 2 c) 3 d) 4

9) $\int_{-1}^{1} (1-|x|)dx =$

a) 1 b) 2 c) $\frac{1}{2}$ d) $\frac{3}{2}$

10) The absolute maximum of the function $f(x)=x^3+2x+2$ on the closed interval $[-2,2]$ is

a) 12 b) 14 c) 5 d) 8

11) The equation of the tangent line to the curve $y=\theta+\cos\theta$, $x=\theta\sin\theta$

when $\theta=\frac{\pi}{2}$ is

a) $y=-x+2$ b) $y=x-1$

c) $y=1$ d) $y=\frac{\pi}{2}$

12) If $\int^{1} f(x)dx=5$, and $\int^{2} f(x)dx=7$, then $\int^{3} f(x)dx=$

$\sqrt{x}(1+\sqrt{x})$ $\qquad$ $1+\sqrt{x}$

4. The solution of the inequality $x+1 \le \frac{2}{x}$ is

a) $(-\infty,0]$ $\qquad$ b) $(-\infty,-2]\cup(0,1]$ $\qquad$ c) $(-\infty,0)\cup(\frac{1}{2},\infty)$ $\qquad$ d) $(-\infty,0)\cup(0,\frac{2}{3}]$

5. The domain of the function $f(x)=\frac{1}{\sqrt{9-x}}$ is :

a) $(-3,\infty)$ $\qquad$ b) $(-\infty,9)$ $\qquad$ c) $(-\infty,3)$ $\qquad$ d) $(-\infty,9)\cup(9,\infty)$

6. The range of the function $f(x)=\frac{1}{\sqrt{9-x}}$ is :

a) $(0,\frac{1}{3}]$ $\qquad$ b) $(-\infty,0)$ $\qquad$ c) $[\frac{1}{3},\infty)$ $\qquad$ d) $(0,\infty)$

7) If $f(x)=\sin 2x$ then $\lim\limits_{h\to 0}\frac{f(\pi+h)-f(\pi)}{h}=$

a) 2 $\qquad$ b) 4 $\qquad$ c) -1 $\qquad$ d) -2

8) $\lim\limits_{x \to 2} \dfrac{x-2}{\sqrt{2x+5}-3} =$

a) 1 b) 2 c) 3 d) 4

9) $\int_{-1}^{1} (1-|x|)\,dx =$

a) 1 b) 2 c) $\frac{1}{2}$ d) $\frac{3}{2}$

10) The absolute maximum of the function $f(x)=x^3+2x-2$ on the closed interval $[-2,2]$ is

a) 12 b) 14 c) 5 d) 8

11) The equation of the tangent line to the curve $y=\theta+\cos\theta$, $x=\theta\sin\theta$

when $\theta=\frac{\pi}{2}$ is

a) $y=-x+2$ b) $y=x-1$

c) $y=1$ d) $y=\frac{\pi}{2}$

12) If $\int_{3}^{2} f(x)\,dx=5$, and $\int_{1}^{2} f(x)\,dx=7$, then $\int_{1}^{3} f(x)\,dx=$

a) -12 b) 2 c) -2 d) 12

12) If $\int_{2}^{1} f(x)dx=5$, and $\int_{1}^{3} f(x)dx=7$, then $\int_{2}^{3} f(x)dx=$

a) -12 b)2 c) -2 d)12

13) $\int_{0}^{1} \sqrt{1-x^2}\,dx =$

a) $\frac{\pi}{4}$ b) $\frac{\pi}{2}$ c) $\frac{\pi}{6}$ d) π

14) If $h(x)=\dfrac{2f(x)}{x+2\cos x}$, $g(x)=\sqrt{x-1}$ and $f(0)=2$ $f'(0)=-1$ then $(g\circ h)'(0)=$

a)0 b)1 c)-2 d) -1

15) If $f(x)=\sin^2(x)-2x$ then f(x) has

a) At least two real roots

b) At least one real roots

c) Exactly one real root

d) Exactly two real roots

math 141

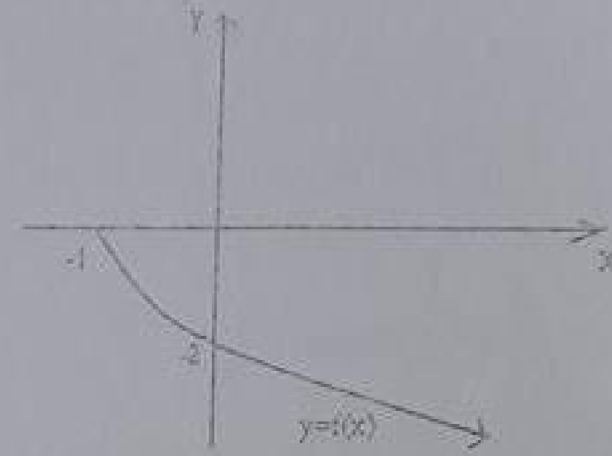
Final Exam — Fall 2011

Name : .... — Number: ....

Instructor of discussion : .... — Section : ....

Question # 1 (56%): Circle the correct answer :

1) The graph on the right represents the graph of the function



a) $f(x)= 1-\sqrt{x-1}$ b) $f(x)= 1-\sqrt{x-1}$

c) $f(x)= 1-\sqrt{x+1}$ d) $f(x)= -2\sqrt{x+1}$

2) $\lim\limits_{x\to 1^-} \dfrac{-2x+5}{x^2-1} =$

a) ∞ b) $-\infty$ c) 0 d) $-\dfrac{2}{5}$

3. $\int \dfrac{dx}{\sqrt{x}(1+\sqrt{x})^2}$

a) $\dfrac{-2}{1+\sqrt{x}}+c$ b) $\dfrac{1}{1+\sqrt{x}}+c$

c) $\dfrac{1}{\sqrt{x}(1+\sqrt{x})}+c$ d) $\dfrac{-1}{1+\sqrt{x}}+c$

4. The solution of the inequality $x+1\le \dfrac{2}{x}$ is

a) $(-\infty,0]$ b) $(-\infty,-2]\cup(0,1]$ c) $(-\infty,0)\cup(\dfrac{1}{2},\infty)$ d) $(-\infty,0)\cup(0,\dfrac{2}{3}]$

23. If $u = 2 - x$, then $\int_0^2 (2-x)^2\,dx =$

(a) $\int_2^0 u^2\,du$

(b) $\int_2^1 u^2\,du$

(c) $\int_0^2 u^2\,du$

(d) $\int_4^2 u^2\,du$

$x = 2$
$u = 2 - 2$

$4 - 4x + x^2$

$4x - 2x^2 + \frac{x^3}{3} \Big|_0^2$

$8 - 8 + \frac{8}{3}$

$A = 2x\,y$

$A = 2x(3 - x^2)$

$A = 6x - 2x^3$

$\frac{dA}{dx} = 6 - 6x^2 = 0$

24. The average value of the function $f(x) = |x|$ over the interval $[-2, 3]$ is

(a) $\frac{5}{2}$

(b) $\frac{1}{2}$

(c) $\frac{13}{2}$

(d) $\frac{13}{10}$

25. Find the function $y = f(x)$ whose curve passes through the point $(1, 4)$ and whose derivative at each point is $3\sqrt{x}$

(a) $y = 2x^{\frac{3}{2}} - 15$

(b) $y = 2x^{\frac{3}{2}} + 2$

(c) $y = \frac{9}{2}x^{\frac{3}{2}} - 35$

(d) $y = \frac{9}{2}x^{\frac{1}{2}} - \frac{1}{2}$

$4 - 4x + x^2$

$4x - 2x^2 + \frac{x^3}{3}$ $\quad \frac{1}{3 - -2}\left(\frac{13}{2}\right)$

$8 - 8 + \frac{8}{3}$

$\frac{13}{10}$

21. Find the derivative of $\int_1^x \frac{\sin t}{t}\,dt$

(a) $\frac{\sin x}{x}$

(b) $\frac{\sin x}{x} - \sin 1$

(c) $\frac{\cos x}{x^2} - \cos 1$

(d) $\frac{x\cos x - \sin x}{x^2}$

22. Find the area of the largest rectangle inscribed in the first quadrant with the left hand corner at the origin and the upper right hand corner on the curve $y = 3 - x^2$

(a) 2

(b) 1

(c) 0

(d) $\sqrt{3}$

$Y = 2$

$x = 1$

$A = 2xY$

$A = 2x(3 - x^2)$

$A = 6x - 2x^3$

$\frac{dA}{dx} = 6 - 6x^2 = 0$

$x = \pm 1$

23. If $u = 2 - x$, then $\int_0^2 (2-x)^2\,dx =$

(a) $\int_2^0 u^2\,du$

(b) $\int_2^1 u^2\,du$

(c) $\int_0^2 u^2\,du$

(d) $\int_1^2 u^2\,du$

$x = 2$

$u = 2 - 2 =$

$4 - 4x + x^2$

$4x - 2x^2 + \frac{x^3}{3}\Big|_0^2$

$8 - 8 + \frac{8}{3}$

24. The average value of the function $f(x) = |x|$ over the interval $[-2, 3]$ is

13. Given that $f(x)$ is continuous and $\int_{-1}^{1} f(x)\,dx = -2$, $\int_{-1}^{3} f(x)\,dx = 6$, then $\int_{1}^{3} f(x)\,dx =$

(a) -4

(b) 8

(c) 4

(d) -8

$\int_{-1}^{3} = \int_{-1}^{1} + \int_{1}^{3}$

$6 = -2 + \int_{1}^{3}$

$\int_{1}^{3} f(x)\,dx = \int$

$\frac{1}{27}$

14. The interval $[\frac{-\pi}{2}, \frac{\pi}{2}]$ is partitioned into n subintervals, let c_k be any point in the kth subinterval whose length is Δx_k, then $\lim_{n\to\infty} \sum_{k=1}^{n} \sin(c_k)\Delta x_k$

(a) 0

(b) 1

(c) -2

(d) 2

$\int_{-1}^{3} f(x)\,dx = \int_{-1}^{1} f(x) + \int_{1}^{3} f(x)$

$6 = -2 + \int_{1}^{3} f(x)$

$\int_{-\pi/2}^{\pi/2} \sin x\,dx$

$-\cos x \Big|_{-\pi/2}^{\pi/2}$

15. $\frac{d}{dx}\left(\int_{0}^{x^2} (1-\sin^2 t)^2\,dt\right) =$

(a) $2x(1-\cos^2(x^2))^2$

(b) $2x\cos^4(x^2)$

(c) $4x(1-\sin^2(x^2))\cos^2(x^2)$

(d) $-8x(1-\sin^2(x^2))\sin(x^2)\cos(x^2)$

$2x(1-\sin^2 x^2)^2$

$(1-\sin^2 x^2)\cos^2 x^2$

$2x($

$(1-\sin^2 x^2)(1-\sin^2 x^2)$

16. $\int \frac{dx}{(2x+3)^2} =$

(a) $\frac{-1}{2x+3} + C$

(b) $\frac{-1}{(2x+3)^2} + C$

(c) $\frac{-1}{3(2x+3)^3} + C$

(d) $\frac{-1}{4x+6} + C$

$\int (2x+3)^{-2}$

$u = 2x+3$

$du = 2\,dx$

$dx = \frac{1}{2}\,du$

$\frac{-1}{2(2x+3)}$

17. $\int \frac{2x+1}{x^3}\,dx =$

(a) $\frac{-2}{3x^3} - \frac{1}{4x^4} + C$

(b) $\frac{-4x-2}{2x^2} + C$

(c) $\frac{-2}{x} - \frac{1}{2x^2} + C$

(d) $\frac{x}{2} + \frac{x^3}{4} + C$

$\int 2x^{-2} + x^{-3}$

$-\frac{2}{x} - \frac{1}{2x^2}$

18. If $f(x) = -x^3 - 3x + 4$, then the linearization of f at $a = 2$ is

(a) $L(x) = -15x - 10$

(b) $L(x) = -15x + 20$

(c) $L(x) = -10x + 5$

(d) $L(x) = -10x - 5$

$f(2) = -8 - 6 + 4 = -10$

$f'(x) = -3x^2 - 3$

$f'(2) = -12 - 3 = -15$

$L(x) = -10 + -15(x-2)$

$= -10 - 15x + 30$

19. One of the following statements is always true

(a) If a function is integrable on $[a,b]$, then it is differentiable on $[a,b]$

(b) $\int_a^b f(x)\,dx$ always exists.

(c) If a function is differentiable on $[a,b]$, then it is integrable over $[a,b]$

(d) The integral of a product of two functions is the product of the integrals of these functions.

11. By integrating with respect to x, the area of the region enclosed by the curves $y = x^2$, and $y = x + 6$ is given by

(a) $\int_{-3}^{2} (x + 6 - x^2)\,dx$

(b) $\int_{-2}^{3} (x + 6 - x^2)\,dx$

(c) $\int_{-2}^{3} (x^2 - x - 6)\,dx$

(d) $\int_{-3}^{2} (x^2 - x - 6)\,dx$

$x^2 = x + 6$

$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

$x = 3,\ x = -2$

12. $\int -2x^2(1-3x^3)^5\,dx =$

(a) $3(1-3x^3)^6 + C$

(b) $\frac{1}{81}(1-3x^3)^6 + C$

(c) $\frac{1}{54}(1-3x^3)^6 + C$

(d) $\frac{1}{27}(1-3x^3)^6 + C$

$u = 1 - 3x^3$

$\frac{du}{dx} = -9x^2$

$dx = \frac{du}{-9x^2}$

$\int -2x^2 u^5 \cdot \frac{du}{-9x^2}$

$\int \frac{2}{9} u^5\,du$

$\frac{2}{9 \cdot 6} u^6$

$\frac{1}{27}$

13. Given that $f(x)$ is continuous and $\int_{-1}^{1} f(x)\,dx = -2$, $\int_{-1}^{3} f(x)\,dx = 6$, then $\int_{1}^{3} f(x)\,dx =$

(a) -4

(b) 8

(c) 4

(d) -3

$\int_{-1}^{3} = \int_{-1}^{1} + \int_{1}^{3}$

$6 = -2 + \int_{1}^{3}$

$\int_{1}^{3} f(x)\,dx = \int$

(d) $L(x) = -10x - 5$

$L(x) = -10 + -15(x-2)$
$= -10 - 15x + 30$

19. One of the following statements is always true

(a) If a function is integrable on $[a, b]$, then it is differentiable on $[a, b]$

(b) $\int_a^b f(x)\,dx$ always exists.

(c) If a function is differentiable on $[a, b]$, then it is integrable over $[a, b]$

(d) The integral of a product of two functions is the product of the integrals of these functions.

20. The solution of the following initial value problem

$$\frac{dy}{dx} = \int_0^x \cos t\,dt, \quad y(0) = -1 \quad \text{is}$$

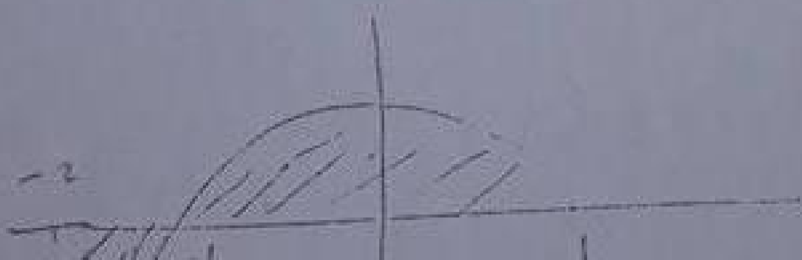
(a) $y = \cos x$

(b) $y = \cos x - 1$

(c) $y = -\cos x - 1$

(d) $y = -\cos x$

$\frac{dy}{dx} = \sin x$
$\int dy = \int \sin x\,dx$
$y = -\cos x + c$
$-1 = -\cos 0 + c$
$-1 = -1 + c$



(d) $\frac{19}{15}\sin^5\left(\frac{\pi x}{?}\right)+C$

9. The area of the region between the curve $y=-x^2+1$ and the x-axis over the interval $[-2,1]$ is

(a) $\frac{8}{3}$

(b) $\frac{16}{3}$

(c) $\frac{4}{3}$

(d) 2

$\int_{-2}^{0} x^2 + \int_{0}^{1} y$

$\frac{8}{3} + \frac{1}{2} \quad \frac{16}{6} + \frac{3}{6}$

$\int_{-1}^{-2} -x^2+1 + \int_{-1}^{1} -x^2+1$

$-\frac{x^3}{3}+x\Big|_{-1}^{-2} + -\frac{x^3}{3}+x\Big|_{-1}^{1}$

$\frac{8}{3}-3-\left[\frac{1}{3}-1\right]+\frac{-1}{3}+1-\left[\frac{1}{3}\right.$

$\frac{8}{3}-3-\frac{1}{3}+1 \quad + \frac{-1}{3}+1-\frac{1}{3}-1$

$\frac{7}{3}-2 \quad +\frac{2}{3} = \frac{6}{3} \quad \frac{9}{3}-\frac{6}{3}$

10. If $f(x)=\begin{cases} x, & x\geq 0, \\ x^2, & x<0 \end{cases}$, then $\int_{-2}^{1} f(x)\,dx =$

(a) $\frac{-5}{3}$

(b) $\frac{-3}{2}$

(c) $\frac{19}{6}$

(d) $\frac{13}{2}$

$\int_{-2}^{1}$

$\int_{-2}^{0} x^2 + \int_{0}^{1} x$

$\frac{x^3}{3}\Big|_{-2}^{0} + \frac{x^2}{2}\Big|_{0}^{1}$

$\frac{8}{3} + \frac{1}{2} = \frac{8}{3}+$

6. $\int_0^3 \sqrt{9-x^2}\,dx$

(a) -18

(b) 18

(c) $\frac{9\pi}{4}$

(d) $\frac{9\pi}{2}$

7. The area between the graphs of the two functions $f(x)$ and $g(x)$ over an interval $[a,b]$ is

(a) $\int_a^b (f(x) - g(x))\,dx$

(b) $\int_a^b (g(x) - f(x))\,dx$

(c) $\int_a^b |f(x) - g(x)|\,dx$

(d) None

$\int 7u^4 \cos\frac{7x}{9} \cdot \frac{9}{7\cos\frac{7x}{9}}$

$9\int u^4\,du$

$u = \sin\frac{7x}{9}$

$\frac{du}{dx} = \frac{7}{9}\cos\frac{7x}{9}$

$dx = \frac{9}{7\cos\frac{7x}{9}}$

$\frac{9}{5}u^5$

$\frac{9}{5}$

8. $\int 7\sin^4\left(\frac{7x}{9}\right)\cos\left(\frac{7x}{9}\right)\,dx =$

(a) $\frac{9}{5}\sin^5\left(\frac{7x}{9}\right) + C$

(b) $\frac{-9}{5}\sin^5\left(\frac{7x}{9}\right) + C$

(c) $\frac{-49}{45}\sin^5\left(\frac{7x}{9}\right) + C$

(d) $\frac{49}{45}\sin^5\left(\frac{7x}{9}\right) + C$

... the curve $y = -x^2 + 1$ and the x-axis over the interval

(d) 0

15. The closest point on the curve $y=\sqrt{x}$ to the point $(2,0)$ is

(a) $(0,0)$

(b) $\left(\frac{3}{2}, \sqrt{\frac{3}{2}}\right)$

(c) $(1,1)$

(d) $\left(\frac{3}{2}, \frac{9}{4}\right)$

16. A particle moves on the curve $y = x^2$ if the x-coordinate of the particle is changing at a rate of 2 cm/sec then the particle distance from the origin at $x=2$ is changing at the rate of

(a) $\frac{18}{\sqrt{5}}$ cm/sec

(b) 8 cm/sec

(c) 4 cm/sec

(d) $10\sqrt{5}$ cm/sec

17. The function $y = \dfrac{\sin x}{x}$ has

(a) A vertical asymptote which is $x = 0$

(b) A horizontal asymptote which $y = 0$

(c) A horizontal tangent at $x = \pi$.

(d) None of the above.

18. $\int_{-1}^{1} x^3 \ldots 1 + x^2 \ldots$

(a) [illegible]

(b) 0

(c) [illegible]

(d) [illegible]

19. One of the following is true

(a) If f is continuous then f is differentiable

(b) If f has a local maximum at x_0 then $f'(x_0) = 0$

(c) If f is differentiable then f is integrable

(d) If $\int_a^b f(x)dx \geq 0$ then $f(x) \geq 0$ for all x in $[a, b]$

20. If $f(x) = \frac{1}{1+x^2}$ then

(a) $0 \leq \int_0^1 f(x)dx \leq$ [illegible]

(b) $0 \leq \int_0^1 f(x)dx \leq 1$

(c) $1 \leq \int_0^1 f(x)dx \leq 2$

(d) none of the above

Circle the most correct answer

1. Find the largest number of

(a) $\int_1^2 \frac{dx}{x^2+1}$

(b) $\int_1^2 \frac{x^2\,dx}{\sqrt{x^2+1}}$

(c) $\int_1^2 \frac{x^2\,dx}{x^2+1}$

(d) $\int_1^2 \frac{dx}{\sqrt{x^2+1}}$

2. $\int_0^{\pi} \sin^2 x\,dx =$

(a) 1

(b) π

(c) $\frac{\pi}{2}$

(d) $\frac{1}{2}$

3. The area bounded by the curves $y = x^2 + 1$, $y = 0$, $x = 1$ and $x = 2$ is

(a) $\frac{7}{3}$

(b) $\frac{16}{3}$

(c) 0

(d) $\frac{10}{3}$

4. If $F(x)$ is an antiderivative of $f(x)$, then

(a) $F'(x) = f(x)$

(b) $F(x) = f(x) + C$

(c) $f'(x) = F(x)$

(d) None

5. $\int \frac{\sec\theta\tan\theta\,d\theta}{\sqrt{1+\sec\theta}} =$

(a) $2\sqrt{1+\sec\theta}+C$

(b) $\frac{1}{\sqrt{1+\sec\theta}}+C$

(c) $\frac{2}{3}(1+\sec\theta)^{\frac{3}{2}}+C$

(d) $-2\sqrt{1+\sec\theta}+C$

12. Use the area to find $\int_0^2 \sqrt{4 - x^2}\,dx =$

(a) π
(b) 2π
(c) $\frac{\pi}{2}$
(d) $\frac{\pi}{4}$

13. If $\varepsilon = 1$ then the value of δ that satisfies the definition of $\lim_{x \to 2} 5 - 2x = 1$ is

(a) 1
(b) $\frac{1}{2}$
(c) $\frac{3}{4}$
(d) 2

14. $\frac{d}{dx} \int_1^{3x} \sqrt{t^2 - 1}\,dt =$

(a) $\sqrt{x^2 - 1}$
(b) $3\sqrt{10}$
(c) $\sqrt{10}$
(d) 0

15. The closest point on the curve $y = \sqrt{x}$ to the point $(2,0)$ is

(d) none of the above

9. If the average value of $y = f(x)$ on the interval $[0,5]$ is 4 then $\int_0^5 f(x)dx =$

(a) 5
(b) 10
(c) 15
(d) 20

10. If $G(x)$ is an antiderivative of $f(x)$ then $G'(x^2)$

(a) $2xf(x^2)$
(b) $2xf(x)$
(c) $\int_0^{x^2} f(t)dt$
(d) $f(x^2)$

11. The domain of $f(x) = \frac{1}{|x| + [x]}$ is

(a) $R - \{0\}$
(b) $R - \{0, 1, 2, \ldots\}$
(c) $R - \{0, -1, -2, \ldots\}$
(d) $R - \{0, \pm 1, \pm 2, \ldots\}$

6. The length of the curve $y = \int_2^x \sqrt{t^2+2t}$ on the interval $2 \le x \le 4$ is

(a) 8

(b) $\frac{9}{2}$

(c) 4

(d) $\frac{1}{2}$

7. The solution set of $\left|\frac{2}{x} - 1\right| < 1$ is

(a) $(0, 1)$

(b) $(1, \infty)$

(c) $(1, 2)$

(d) none of the above

$-1 < \frac{2}{x} - 1 < 1$

$\frac{2}{x} - 1$

8. The equation of the normal to the a curve $f(x) = x^2 - x + 1$ at $x = 1$ is

(a) $y = -x$

(b) $y = x$

(c) $y = -x + 2$

(d) none of the above

... $f(x)$ on the interval $[0, 5]$ is 4 then $\int^5 f(x)dx =$

(13) If $f(x) = 3\log_2 x$, then $f^{-1}(x) =$

(a) $2^{x/3}$

(b) 2^{3x}

(c) $e^{x/3}$

(d) e^{3x}

$y = \frac{\ln x^3}{\ln 2}$

(14) If $y = \ln(\csc\theta) + \ln(\sin^2\theta)$, then $\frac{dy}{d\theta} =$

(a) $\csc\theta$

(b) $-\csc\theta$

(c) $\cot\theta$

(d) $-\cot\theta$

$\ln(\cos\theta \sin^2\theta)$

$\ln(\cos\theta(1-\cos^2\theta))$

$\ln(\cos\theta - \cos^3\theta)$

$\frac{\ln\cos\theta}{\ln\cos^3\theta}$

$\frac{\sin\theta - 3\cos^2\theta\sin\theta}{\cos\theta - \cos^3\theta}$

(15) $\int x^2 \cosh x \, dx =$

(a) $x^2 \sinh x + 2x\cosh x - 2\sinh x + c$

(b) $-x^2 \sinh x + 2x\cosh x - 2\sinh x + c$

(c) $x^2 \sinh x - 2x\cosh x + 2\sinh x + c$

(d) $-x^2 \sinh x - 2x\cosh x - 2\sinh x + c$

$u = x^2 \quad dv = \cosh x$

$du = 2x\,dx \quad v = \sinh x$

$x^2 \sinh x - \int 2x \sinh x \, dx$

(16) The area between $f(x) = 3^x \ln 3$ and the x-axis on the interval $[0, 1]$ is

(a) 3

(b) 2

(c) $3\,(\ln 3)^2$

(d) $2\,(\ln 3)^2$

$\int 3^x = \frac{3^x}{\ln 3}\Big|_0^1 = \frac{3^1}{\ln 3} - \frac{3^0}{\ln 3} = \frac{3-1}{\ln 3} =$

1. When the graph of $y = \sin(2x)$ is shifted $\dfrac{\pi}{2}$ units to the right and 3 units up then the resulting graph is described by

(a) $y = \sin(2x + \pi) + 3$

(b) $y = \sin(2(x - \frac{\pi}{2})) + 3$

(c) $y = 3\sin(2(x + \frac{\pi}{2}))$

(d) $y = \sin(2x + \frac{\pi}{2}) + 3$

2. $\displaystyle\lim_{x \to 0^-} \frac{x}{[x]} =$

(a) 0

(b) ∞

(c) $-\infty$

(d) doesn't exist

3. $\displaystyle\lim_{x \to 3} \frac{x^2 - 9}{\sqrt{x^2 + 7} - 4} =$

(a) 4

(b) 12

(c) 8

(d) 16

4. $\frac{d}{dx}(\sin(\cos 2x)) =$ $\quad = \cos(\cos 2x)(-4\sin 2x)$

(a) $2\cos(\sin 2x)$

(b) $-2(\cos 2x)\cos$

(c) $2\sin(\cos 2x)$

(d) $-2(\sin 2x)\cos(\cos 2x)$

5. The slope of the curve $x^2 + \sin xy = 1 + \sqrt{\frac{3}{2}}$ at the point $(1, \frac{\pi}{3})$ is

$2x + \cos xy\,(x y' + y \cdot 1) = 0$

(a) $\frac{-12-\pi}{3}$

(b) 2

(c) 0

(d) none of the above

$-2 = \frac{1}{2} y' + \frac{1}{2} \times \frac{\pi}{3}$

$y' = 2(-2 - \frac{\pi}{6})$

(9) $\int 15 \sin^2 x \cos^3 x \, dx =$

(a) $5\cos^3 x + 3\cos^5 x + c$

(b) $5\cos^3 x - 3\cos^5 x + c$

(c) $5\sin^3 x - 3\sin^5 x + c$

(d) $5\sin^3 x + 3\sin^5 x + c$

(10) Given the functions, $x^{50}, \ln(x^{10}), (\ln x)^x, 2^x, \sqrt[3]{x}$, the slowest function as $x \to \infty$ is

(a) $\sqrt[3]{x}$

(b) $\ln(x^{10})$

(c) $(\ln x)^x$

(d) x^{50}

(11) Using the trigonometric substitution $x = 2\tan\theta$, the integral $\int \frac{x^3\,dx}{\sqrt{x^2+4}} =$

(a) $\int 8\tan^3\theta \sec\theta \, d\theta$

(b) $\int \frac{8\tan^3\theta \, d\theta}{\sec\theta}$

(c) $\int 16\tan^3\theta \sec^2\theta \, d\theta$

(d) $\int \frac{4\tan^3\theta \, d\theta}{\sec\theta}$

(12) $\int \frac{4\,dx}{(x+1)\sqrt{(x+1)^2-4}} =$

(a) $2\sec^{-1}|x+1| + c$

(b) $\sec^{-1}|x+1| + c$

(c) $2\sec^{-1}\left|\frac{x+1}{2}\right| + c$

(d) $\sec^{-1}\left|\frac{x+1}{2}\right| + c$

(5) If $f(x)$ passes through the point $(2,4)$ with slope $= \frac{1}{3}$, then $(f^{-1})'(4) =$

(a) 3

(b) 2

(c) $\frac{1}{3}$

(d) $\frac{1}{2}$

$f'(x) = \frac{1}{3}$

$\frac{1}{\frac{1}{3}}$

$= 3$

(6) $\tan(\cos^{-1}(\frac{-1}{2})) =$

(a) $-\sqrt{3}$

(b) $\sqrt{3}$

(c) $-\frac{1}{\sqrt{3}}$

(d) $\frac{1}{\sqrt{3}}$

(7) $\lim_{x \to 0} \frac{1 + 6x - \operatorname{sech} x}{x + \sinh x} = \frac{6 - \operatorname{sech} x \tanh x}{1 + \cosh x} = \frac{6 - 0}{1 + 1} = 3$

(a) 2

(b) 0

(c) 6

(d) 3

(8) $\int_5^6 \frac{dx}{x^2 - 7x + 12} = \frac{A}{(x-4)} + \frac{B}{(x-3)}$

$A(x-3) + B(x-4) = 1$

$Ax - 3A + Bx - 4B = 1$

$A + B = 0$

$-3A - 4B = 1$

(a) $\ln 4 - \ln 3$

(b) $\ln 2 - \ln 3$

(c) $2\ln 2 - 3\ln 3$

(d) $2 \ln 2$